EXPONENTS AND RADICALS

$x^m x^n = x^{m+n}$ $\qquad \dfrac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$ $\qquad x^{-n} = \dfrac{1}{x^n}$

$(xy)^n = x^n y^n$ $\qquad \left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$ $\qquad \sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

SPECIAL PRODUCTS

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

FACTORING FORMULAS

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$.

$|x| < a$ means $-a < x < a$.

$|x| > a$ means $x > a$ or $x < -a$.

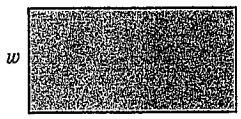
GEOMETRIC FORMULAS

Formulas for area A, perimeter P, circumference C, volume V:

Rectangle

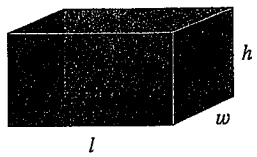
$A = lw$

$P = 2l + 2w$



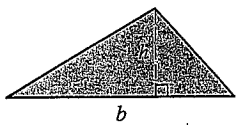
Box

$V = lwh$



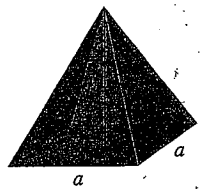
Triangle

$A = \frac{1}{2}bh$



Pyramid

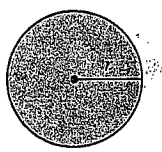
$V = \frac{1}{3}ha^2$



Circle

$A = \pi r^2$

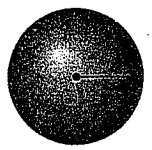
$C = 2\pi r$



Sphere

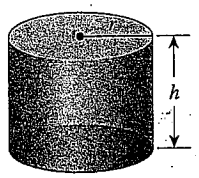
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$



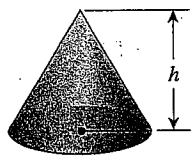
Cylinder

$V = \pi r^2 h$



Cone

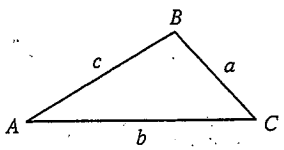
$V = \frac{1}{3}\pi r^2 h$



HERON'S FORMULA

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$



DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m: $y - y_1 = m(x - x_1)$

Slope-intercept equation of line with slope m and y-intercept b: $y = mx + b$

Two-intercept equation of line with x-intercept a and y-intercept b: $\frac{x}{a} + \frac{y}{b} = 1$

LOGARITHMS

$y = \log_a x$ means $a^y = x$

$\log_a a^x = x$ $\qquad$ $a^{\log_a x} = x$

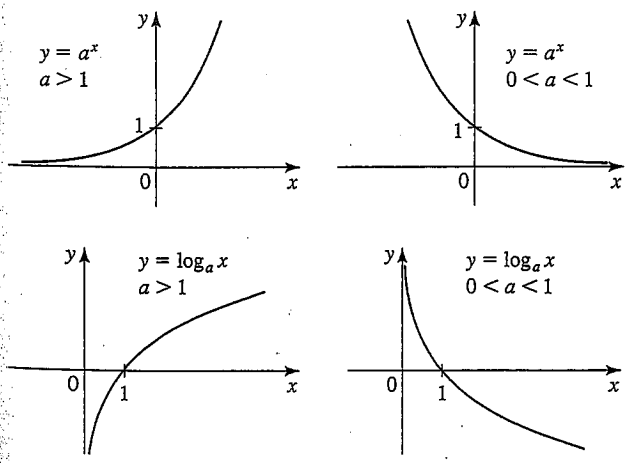
$\log_a 1 = 0$ $\qquad$ $\log_a a = 1$

$\log x = \log_{10} x$ $\qquad$ $\ln x = \log_e x$

$\log_a xy = \log_a x + \log_a y$ $\qquad$ $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

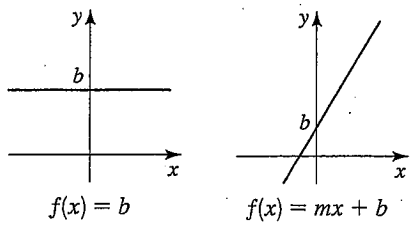
$\log_a x^b = b \log_a x$ $\qquad$ $\log_b x = \frac{\log_a x}{\log_a b}$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

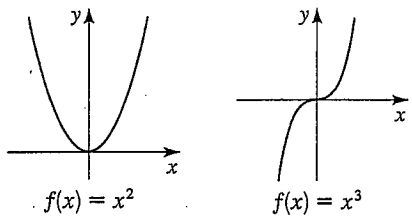


GRAPHS OF FUNCTIONS

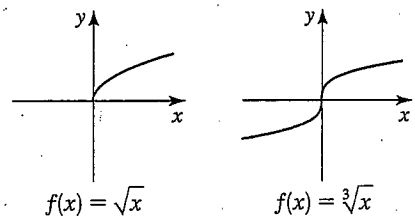
Linear functions: $f(x) = mx + b$



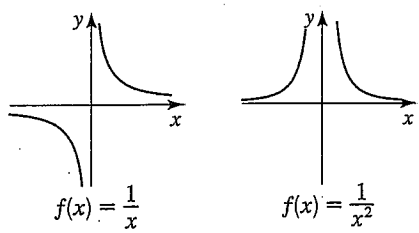
Power functions: $f(x) = x^n$



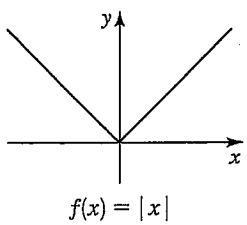
Root functions: $f(x) = \sqrt[n]{x}$



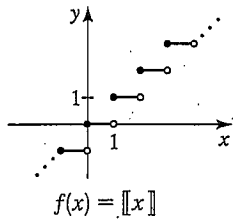
Reciprocal functions: $f(x) = 1/x^n$



Absolute value function



Greatest integer function



SEQUENCES AND SERIES

Arithmetic

$a, a + d, a + 2d, a + 3d, a + 4d, \ldots$

$a_n = a + (n - 1)d$

$S_n = \sum_{k=1}^{n} a_k = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$

Geometric

$a, ar, ar^2, ar^3, ar^4, \ldots$

$a_n = ar^{n-1}$

$S_n = \sum_{k=1}^{n} a_k = a\frac{1 - r^n}{1 - r}$

If $|r| < 1$, then the sum of an infinite geometric series is

$$S = \frac{a}{1 - r}$$

THE BINOMIAL THEOREM

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

FINANCE

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded n times per year.

Amount of an annuity

$$A_f = R\frac{(1 + i)^n - 1}{i}$$

where A_f is the final amount, i is the interest rate per time period, and there are n payments of size R.

Present value of an annuity

$$A_p = R\frac{1 - (1 + i)^{-n}}{i}$$

where A_p is the present value, i is the interest rate per time period, and there are n payments of size R.

Installment buying

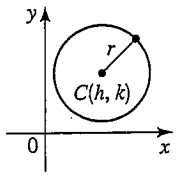
$$R = \frac{iA_p}{1 - (1 + i)^{-n}}$$

where R is the size of each payment, i is the interest rate per time period, A_p is the amount of the loan, and n is the number of payments.

CONIC SECTIONS

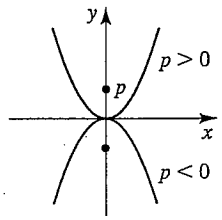
Circles

$(x - h)^2 + (y - k)^2 = r^2$



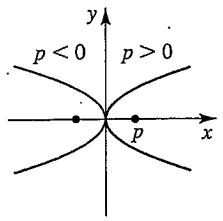
Parabolas

$x^2 = 4py$

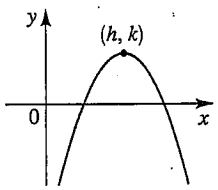


Focus $(0, p)$, directrix $y = -p$

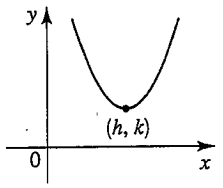
$y^2 = 4px$



Focus $(p, 0)$, directrix $x = -p$



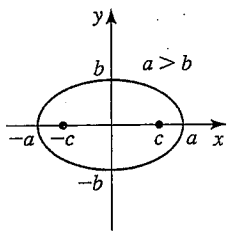
$y = a(x - h)^2 + k$,
$a < 0, \quad h > 0, \quad k > 0$



$y = a(x - h)^2 + k$,
$a > 0, \quad h > 0, \quad k > 0$

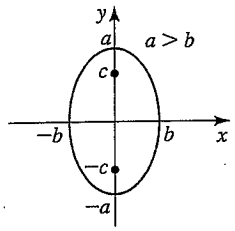
Ellipses

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

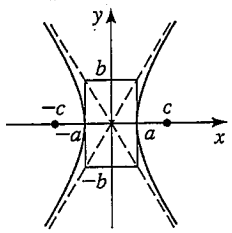
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$



Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

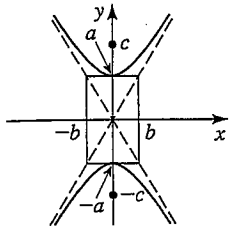
Hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

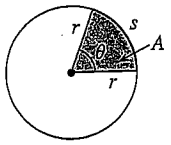
$-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

ANGLE MEASUREMENT

π radians $= 180°$



$1° = \frac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \frac{180°}{\pi}$

$s = r\theta \qquad A = \frac{1}{2}r^2\theta \quad (\theta \text{ in radians})$

To convert from degrees to radians, multiply by $\frac{\pi}{180}$.

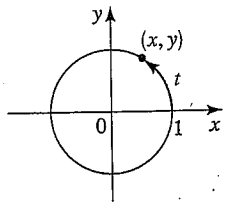
To convert from radians to degrees, multiply by $\frac{180}{\pi}$.

TRIGONOMETRIC FUNCTIONS OF REAL NUMBERS

$\sin t = y \qquad \csc t = \frac{1}{y}$

$\cos t = x \qquad \sec t = \frac{1}{x}$

$\tan t = \frac{y}{x} \qquad \cot t = \frac{x}{y}$

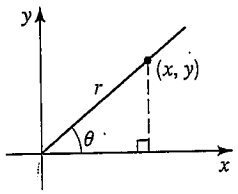


TRIGONOMETRIC FUNCTIONS OF ANGLES

$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$

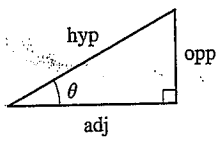


RIGHT ANGLE TRIGONOMETRY

$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$

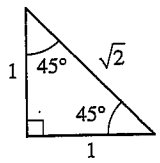
$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$

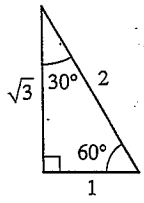


SPECIAL VALUES OF THE TRIGONOMETRIC FUNCTIONS

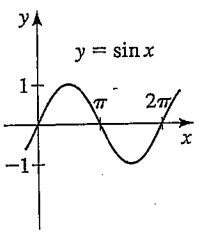
θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	$\pi/2$	1	0	—
180°	π	0	−1	0
270°	$3\pi/2$	−1	0	—

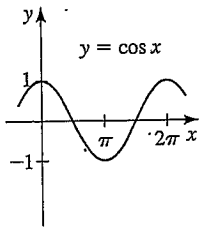
SPECIAL TRIANGLES

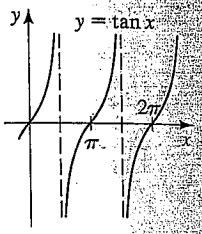


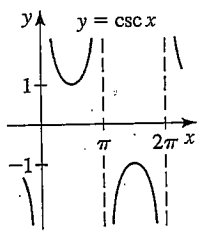


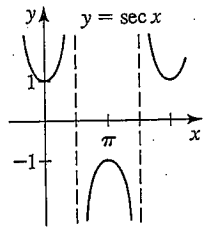
GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

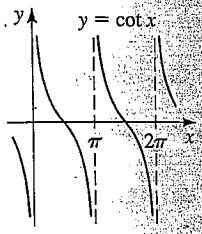






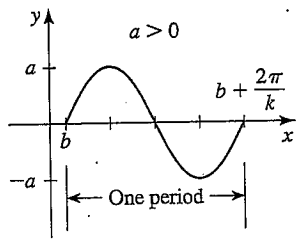


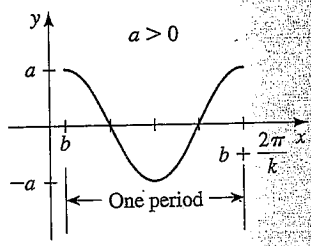




SINE AND COSINE CURVES

$y = a \sin k(x - b) \quad (k > 0)$ $\qquad$ $y = a \cos k(x - b) \quad (k > 0)$

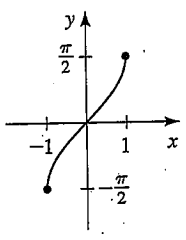




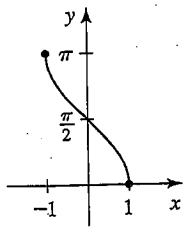
amplitude: $|a|$ $\qquad$ period: $2\pi/k$ $\qquad$ phase shift: b

GRAPHS OF THE INVERSE TRIGONOMETRIC FUNCTIONS

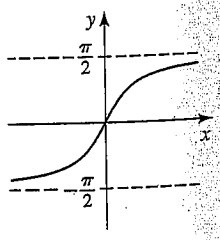
$y = \sin^{-1}x$



$y = \cos^{-1}x$



$y = \tan^{-1}x$



FUNDAMENTAL IDENTITIES

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

REDUCTION IDENTITIES

$$\sin(x + \pi) = -\sin x \qquad \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos(x + \pi) = -\cos x \qquad \cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\tan(x + \pi) = \tan x \qquad \tan\left(x + \frac{\pi}{2}\right) = -\cot x$$

ADDITION AND SUBTRACTION FORMULAS

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \qquad = 1 - 2\sin^2 x$$

FORMULAS FOR REDUCING POWERS

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

PRODUCT-TO-SUM AND SUM-TO-PRODUCT IDENTITIES

$$\sin u \cos v = \tfrac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \tfrac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \tfrac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \tfrac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\sin x + \sin y = 2 \sin\frac{x + y}{2} \cos\frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos\frac{x + y}{2} \sin\frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos\frac{x + y}{2} \cos\frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin\frac{x + y}{2} \sin\frac{x - y}{2}$$

THE LAWS OF SINES AND COSINES

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$