Probability Cheatsheet v2.0

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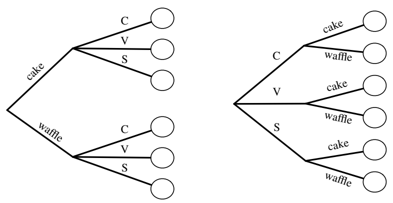
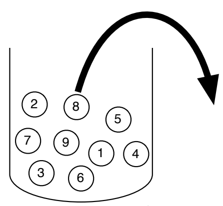
Counting

Multiplication Rule



Let's say we have a compound experiment (an experiment with multiple components). If the 1st component has n_1 possible outcomes, the 2nd component has n_2 possible outcomes, $\dots$, and the rth component has n_r possible outcomes, then overall there are $n_1 n_2 \dots n_r$ possibilities for the whole experiment.

Sampling Table



The sampling table gives the number of possible samples of size k out of a population of size n, under various assumptions about how the sample is collected.

	Order Matters	**Not Matter**
With Replacement	n^k	$\binom{n+k-1}{k}$
Without Replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Naive Definition of Probability

If all outcomes are equally likely, the probability of an event A happening is:

$$P_{\text{naive}}(A) = \frac{\text{number of outcomes favorable to } A}{\text{number of outcomes}}$$

Thinking Conditionally

Independence

Independent Events A and B are independent if knowing whether A occurred gives no information about whether B occurred. More formally, A and B (which have nonzero probability) are independent if and only if one of the following equivalent statements holds:

$$P(A \cap B) = P(A)P(B)$$
$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

Conditional Independence A and B are conditionally independent given C if $P(A \cap B|C) = P(A|C)P(B|C)$. Conditional independence does not imply independence, and independence does not imply conditional independence.

Unions, Intersections, and Complements

De Morgan's Laws A useful identity that can make calculating probabilities of unions easier by relating them to intersections, and vice versa. Analogous results hold with more than two sets.

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Joint, Marginal, and Conditional

Joint Probability $P(A \cap B)$ or $P(A, B)$ – Probability of A and B.

Marginal (Unconditional) Probability $P(A)$ – Probability of A.

Conditional Probability $P(A|B) = P(A, B)/P(B)$ – Probability of A, given that B occurred.

Conditional Probability *is* Probability $P(A|B)$ is a probability function for any fixed B. Any theorem that holds for probability also holds for conditional probability.

Probability of an Intersection or Union

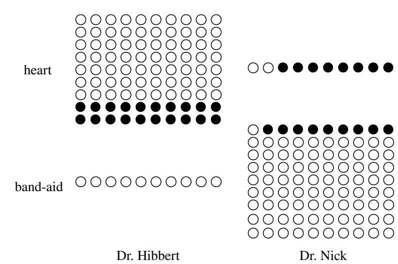
Intersections via Conditioning

$$P(A, B) = P(A)P(B|A)$$
$$P(A, B, C) = P(A)P(B|A)P(C|A, B)$$

Unions via Inclusion-Exclusion

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Simpson's Paradox



It is possible to have

$$P(A \mid B, C) < P(A \mid B^c, C) \text{ and } P(A \mid B, C^c) < P(A \mid B^c, C^c)$$
$$\text{yet also } P(A \mid B) > P(A \mid B^c).$$

Law of Total Probability (LOTP)

Let $B_1, B_2, B_3, \dots B_n$ be a *partition* of the sample space (i.e., they are disjoint and their union is the entire sample space).

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_n)P(B_n)$$
$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

For **LOTP with extra conditioning**, just add in another event C!

$$P(A|C) = P(A|B_1, C)P(B_1|C) + \dots + P(A|B_n, C)P(B_n|C)$$
$$P(A|C) = P(A \cap B_1|C) + P(A \cap B_2|C) + \dots + P(A \cap B_n|C)$$

Special case of LOTP with B and B^c as partition:

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$
$$P(A) = P(A \cap B) + P(A \cap B^c)$$

Bayes' Rule

Bayes' Rule, and with extra conditioning (just add in C!)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A|B, C) = \frac{P(B|A, C)P(A|C)}{P(B|C)}$$

We can also write

$$P(A|B, C) = \frac{P(A, B, C)}{P(B, C)} = \frac{P(B, C|A)P(A)}{P(B, C)}$$

Odds Form of Bayes' Rule

$$\frac{P(A|B)}{P(A^c|B)} = \frac{P(B|A)}{P(B|A^c)} \frac{P(A)}{P(A^c)}$$

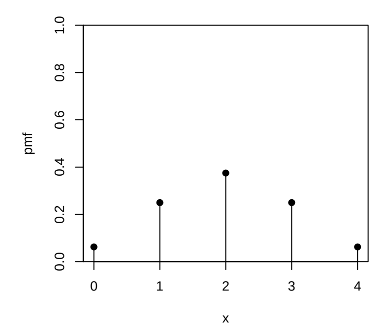
The *posterior odds* of A are the *likelihood ratio* times the *prior odds*.

Random Variables and their Distributions

PMF, CDF, and Independence

Probability Mass Function (PMF) Gives the probability that a *discrete* random variable takes on the value x.

$$p_X(x) = P(X = x)$$



The PMF satisfies

$$p_X(x) \geq 0 \text{ and } \sum_x p_X(x) = 1$$